Basic Introduction

Here we introduce and show you how to use a deductive system for sentential or propositional logic. You will learn how to use a deductive system either with pencil and paper or with the proof-builder included in *BlueStorm: The Logic Course*. We will presuppose that you are familiar with both formal languages and certain basic semantic concepts. You can familiarize yourself with both of these either by reading the text *The Logic Course* or by using *BlueStorm*.

The language we are using utilizes

A, B, ……….. Z

as atomic or basic sentences or well-formed formulae. In order to assure that we have a an infinite supply of basic sentences we also allow that each of the above when numerically subscripted is a basic sentence as well. Thus P1 is, for example, a basic sentence. Our connectives are:

&, v, ->, <->, ~

Parentheses are used in the standard way. We utilize these to form **complex** or **compound sentences**. The first four are called **binary connectives** since one uses them to form a new sentence by placing them between two previously constructed sentences. The outermost pair of parentheses, if there is such, is typically deleted. A sentence whose main connective is:

‘&’	is a conjunction (the sentences from which it is formed are conjuncts)
‘v’	is a disjunction (the sentences from which it is formed are disjuncts)
‘->’	is a conditional (the sentence on the left is the antecedent, on the right the consequent)
‘<->’	is a biconditional (no special name for the sentences from which it is formed)
‘~’	is a negation

We will use lowercase letters as metavariables. Thus we will use ‘p & q’ if we wish to talk about any conjunction. On occasion we will use * as a metavariable for connectives. Thus ‘p * q’ could be used if we wished to talk about any sentence that was a conjunction, disjunction, conditional or biconditional. Each compound or complex sentence will be either a conjunction, a disjunction, a conditional, a biconditional or a negation. It is very important to pay attention to the form or the structure of a compound sentence. Thus:

~P & Q

is a conjunction, not a negation. The left conjunct, one of the subsentences, is a negation but the sentence as a whole is not. But:

~(P & Q)

is a negation. It is the negation of a conjunction. You will avoid many errors if you carefully attend to the form of the sentences with which you are working.

Arguments are truth-functionally valid if there is no assignment of truth-values in which all the premises are true and the conclusion is false. We may also say that the conclusion follows truth-functionally from the premises. A sentence that is true in all assignments of truth-values is said to be logically true or a tautology, one false in all assignments is said to be logically false or a contradiction. A set of sentences is truth-functionally consistent if and only if there is at least one assignment of truth values in which all the sentences are true (henceforth simply consistent).

The goal of our system is to enable us to move, via the application of rules, from a set of premises to any conclusion that does indeed follow truth-functionally from those premises. A sentence p follows truth-functionally from a set of sentences if and only if the argument with those sentences as premises and p as the conclusion is truth-functionally valid. Henceforth we will simply say ‘follows from’ rather than ‘follows truth-functionally from’. A system that in principle enables us to move, via the application of rules, from a set of premises to any conclusion that does indeed follow from those premises is said to be **complete**. But a system should have another virtue, too. As well as being complete, it should be **sound**. That is, if you follow the rules of the system you will not be able to reach a conclusion that does not follow from the set of premises with which you are working. We will here develop a complete and sound system that you will, with modest effort, be able to use.

Certain arguments are truth-functionally valid, certain others truth-functionally invalid (henceforth simply valid or invalid). Note that to say that an argument is valid is the same as saying that the conclusion follows from the premises. Suppose you are presented with an argument that you do not know to be valid. Can you find out whether the argument is valid or invalid by using a deductive system? The answer is yes and no. It is yes in the sense that if you do reach the conclusion by using the deductive system correctly, you know (since the system is a sound one) that the argument in question is a valid one. But what if you are unable to reach the conclusion? Have you established that the argument is invalid? The answer to this is no. You may simply have overlooked a way to reach the conclusion. Of course in certain cases you will, by noting why you are unable to reach a conclusion, be able to “see” that an argument is invalid. But the system itself does not tell you this.

Why do we have or need deductive systems? After all, there are other ways to determine whether an argument is valid or invalid. We could use truth tables, for example. But there are various reasons why deductive systems are of value. First, as we shall see when we study arguments that are valid but not truth-functionally valid, there are cases in which there is no full-fledged analog of truth-tables, that is, no “mechanical” means of always determining in a finite number of steps whether or not an argument is valid. When we come to study such arguments we will rely upon a deductive system. Second, in much of our actual reasoning we often move from premises to a conclusion by the use of rules of inference. A system of the sort we are developing is sometimes called a system of "natural deduction". It is not, of course, true that it is completely "natural" in the sense of exactly mirroring our ordinary reasoning. But it is nonetheless related to the reasoning that we use in ordinary life. By mastering the deductive system we can hopefully develop the ability to reason in a more disciplined fashion. We may make fewer mistakes and may be better able to see that certain reasoning does involve mistakes.

When using our system we typically proceed in the following way. An argument begins with certain premises. We first list those premises. Each one will be entered on a separate numbered line. For example, if our premises are

P v Q and ~P

the opening of what we shall speak of as a **derivation** will look like this:

1. P v Q	Premise
2. ~P	Premise

If an argument is valid, we should be able to move from the premises to the conclusion in a sequence of steps. Each step, each additional line, will be one that the rules of our deductive system will allow us to add. If we reach the desired conclusion by the correct application of the rules, we shall say that we have a derivation or proof of the conclusion from those premises, or, to be slightly more formal, a derivation of our conclusion from that set of premises.

Q

does in fact follow from these premises. We should be able to obtain it as a line by applying the rules of our system. In order to keep track of where a line comes from, we enter on the right an account -- we shall call it a justification -- that tells us how the line was derived, where it came from. In this case our initial two lines are justified in the sense that they are premises.

You will frequently see the symbol ‘|-’ in such contexts as:

P, Q |- P & Q

This says that ‘P & Q’ is derivable from the premise set ‘P’, ‘Q’. You will frequently be asked, for example, to:

Show that ~P v Q, P |- Q

In such a case you are being asked to construct a derivation of the sentence on the right from those on the left. Given that our system is sound you can construct a derivation to show that the argument is valid -- that the conclusion follows from those premises.

This manual is divided into three main sections. In the first section we introduce what we shall call the basic rules of the system. These do not require the use of assumptions (we will discuss assumptions in the second section). In the second section we will introduce the final two rules. These rules are different conceptually from the basic rules and will require the use of assumptions. In the final section we will turn to various special topics.