Introduction to Assumptions

While our current system enables us to do many derivations it is not yet a complete system. For example, it follows from p -> q and q -> r that p -> r. But we do not yet have the means to show this. Similarly ~(p v q) |- ~p & ~q, but, again, we have as yet no way to show it. In order to have available a complete system, we will here introduce what we shall call modes of proof. These modes of proof will involve utilizing assumptions. Let us now look at a common argument that involves, at least when we defend it, utilizing an assumption. You have said the following:

> If the switch is turned to the left when it is in the middle, the light dims. If the light dims, then the display can't be seen. So, if the switch is turned to the left when it is in the middle, then the display can't be seen.

Someone challenges your conclusion. One way in which you might defend it is as follows:

> Look you are granting that (1) if the (S)witch is turned to the left when it is in the middle, the (L)ight dims. You are also granting that (2) if the light dims, then the (D)isplay can't be seen. Now suppose that the switch is turned to the left when it is in the middle. It follows, given (1), that the light dims. Since the light dims it follows, given (2), that the display can't be seen. So if the switch is turned to the left when it is in the middle, then the display can't be seen.

Here you are in effect using a mode of proof that we shall call conditional introduction (->I). (It is sometimes called 'conditional proof'.) After you make an assumption (assumptions are often introduced by 'suppose'), you then make a claim that you have reached a conclusion that depends only upon the premises. Here is the way we will set up this ->I:

1.	S -> L	Premise
2.	L -> ~D	Premise

As per usual we initiate a derivation by making the premises the initial lines. Next we will introduce our assumption:

1. S -> L		Premise
2. L -> ~D		Premise
3.	S	A

Line 3 is "justified" by way of being an assumption — for short we use A. Notice that here, as we will always, we indent any assumption one column to the right of the line immediately above it. This is simply a way to make it visually clear that we have made an assumption. We will now continue this derivation in the way characteristic of our particular deductive systems:

1. S -> L		Premise
2. L -> ~D		Premise
3.	S	A
4.	L	1, 3 ->E

Note that line 4 has arrived by an application of ->E. The assumption line 3 is the initial line of what we will call a subderivation. Let us continue our derivation:

1. S -> L		Premise
2. L -> ~D		Premise
3.	S	A
4.	L	1, 3 ->E
5.	~D	2, 4 ->E

We now have, as the last line underneath the conditional, the consequent of the conditional that we wish to obtain. At this point we can end or terminate our subderivation as follows:

1.	S -> L		Premise
2.	L -> ~D		Premise
3.		S	A
4.		L	1, 3 ->E
5.		~D	2, 4 ->E
6.	S -> ~D		3-5 ->I

We have obtained line 6 by an application of our ->I rule. Note that line 6 is no longer indented. At line 6 the assumption has been **discharged**. When we apply ->I, the line we add will always be one column to the left of the subderivation that is discharged. The particular subderivation, lines 3-5, is **terminated**. **No line within a terminated subderivation may be appealed to at any later point in the derivation**. We call such lines **inaccessible**. As a helpful visual cue, we will place an * just before the justification of any line that has become inaccessible. So this is the final appearance of this derivation:

1.	S -> L		Premise
2.	L -> ~D		Premise
3.		S	* A
4.		L	* 1, 3 ->E
5.		~D	* 2, 4 ->E
6.	S -> ~D		3 - 5 ->I

We have now shown that S -> L, L -> ~D I- S -> ~D. Keep in mind that indented lines or lines marked off with an * are not guaranteed to follow from the premises of the derivation. ~D, for example, does not follow from premises 1 and 2. It depends on the assumption. No inaccessible line can be appealed to or reiterated. Notice that the justification for line 6 says 3 - 5 rather than 3, 7. This indicates that you are appealing to the subderivation as a whole.

On occasion it will be useful to repeat or reiterate a line. So we will introduce a reiteration rule (Reit for short).

Reit
From p
To p

Reit, like any of our rules, can only be used when the line to which you appeal is an accessible line. We will later look at some examples where the use of reiteration is useful.

Consider the following argument:

> It's not the case that either (A)ffirming the consequent is valid or (D)enying the antecedent is valid. So denying the antecedent is not valid.

Someone again (implausibly, I admit) challenges your conclusion. You might respond as follows:

> You grant that (1) it's not the case that either (A)ffirming the consequent is valid or (D)enying the antecedent is valid. Now suppose that it is true that denying the antecedent is valid. Then it is true that either affirming the consequent is valid or denying the antecedent is valid. But (1) tells us that that is false. So the supposition that denying the antecedent is valid is also false. That is, denying the antecedent is not valid.

Here you are using a mode of proof that we shall call negation introduction (~I). (It is sometimes called 'indirect proof' or 'reductio ad adsurdam'.) The core idea is that if an assumption, given your premises, leads to a contradiction then the assumption must, given your premises, be false. But if the assumption, given your premises, must be false, the negation of the assumption, given your premises, must be true. In our system you can only apply negation introduction if you have, as the last line in a subderivation, a line that is an instance of either p & ~p or ~p & p, that is if we have an explicit contradiction. Here is a way to show the above argument valid in our system:

1.	~(A v D)	Premise
2.	D	* A
3.	A v D	* 2 vI
4.	(A v D) & ~(A v D)	* 1,3 &I
5.	~D	2-4 ~I

Notice that the line 4 here is an explicit contradiction. So we can utilize ~I if we wish. Any application of ~I will give us only the negation of the assumption with which we began. And that is the conclusion that we wish to reach here.

We will also introduce a negation elimination rule (~E) If the sentence p we wish to derive is not a negation, then we assume its negation ~p and, once we have obtained an explicit contradiction, apply ~E. The application of ~E gives us p. Note that ~E is in a certain way redundant. We could assume ~p and then use ~I to obtain ~~p, then obtaining p by applying ~~E.

Be very careful not to confuse ~~I and ~~E with ~I and ~E. The former allow us to introduce or eliminate double negations. The latter are modes of proof that involve the use of assumptions.