Working with Conditional Introduction

In “Introduction to Assumptions” we provided a brief look at our ->I rule. Here we will examine it in more detail. Schematically we can exhibit it as follows:

	->I	
	.	
	.	
m	p	* A
	.	
	.	
m + n	q	*
m + n + 1	p -> q	m - m+n ->I

When you introduce any assumption you should indent one column. When you discharge an assumption you move one column to the left of the assumption that is discharged. Once an assumption is discharged the lines in the subderivation are, as indicated by the *, inaccessible.

We cannot here undertake a formal proof of the soundness of ->I, that is, a proof that it will not enable us to derive sentences that do not follow from the premises that we have. But here is the basic reasoning. Look at the exhibit above: p will be either true or false. Where p is false, the conditional p -> q will be true. So the only case in which we might have true premises and a false conclusion is the case in which p is true. But we assumed that p is true. And then we showed that where p is true, given the premises, q must also be true. So there is no case where the premises are true and p -> q is false.

As a general rule it is best to try to establish a conditional by setting up a conditional proof. Here is an example wherein we show that ~P v Q |- P -> Q:

1.	~P v Q	Premise
2.	P	* A
3.	~~P	* 2 ~~I
4.	Q	* 1, 3 DS
5.	P -> Q	2-4 ->I

In this particular case we had a specific goal. We wanted to show that ~P v Q |- P -> Q. But you can terminate and discharge an ->I at any point. For example, the following is a perfectly respectable derivation:

1.	~P v Q	Premise
2.	P	* A
3.	~~P	* 2 ~~I
4.	P -> ~~P	2-3 ->I

Here we have shown that that ~P v Q |- P -> ~~P. A decision to terminate and discharge is made on the basis of the goal that one has.

In the preceding section we introduced a rule Reit. Here is a case in which we could use it. We will show that Q I- P -> Q:

1.	Q	Premise
2.	P	* A
3.	Q	* 1 Reit
4.	P -> Q	2-3 ->I

When we apply ->I we can only obtain a conditional whose antecedent is the assumption and whose consequent is the last line of the subderivation.

Here is another derivation, in which we show that P & (~P v (~A & ~B)) |- A -> D:

1. P & (~P v (~A & ~B))	Premise
2. P	1 &E
3. ~P v (~A & ~B)	1 &E
4. ~~P	2 ~~I
5. ~A & ~B	3, 4 DS
6. ~A	5 &E
7. A	* A
8. A v D	* 7 vI
9. D	* 6, 8 DS
10. A -> D	7-9 ->I

Notice that here I applied basic rules before making my assumption. I did not use &E to obtain ~B since I saw that I did not need to do so. But even if I had, I would still have a correct derivation:

1. P & (~P v (~A & ~B))	Premise
2. P	1 &E
3. ~P v (~A & ~B)	1 &E
4. ~~P	2 ~~I
5. ~A & ~B	3, 4 DS
6. ~B	5 &E
7. ~A	5 &E
8. A	* A
9. A v D	* 8 vI
10. D	* 7,9 DS
11. A -> D	8-10 ->I

This is one line longer, but it is a correct derivation. Let us do two more derivations before turning to ~I. Here we show that (A <-> ~~B) |- (~B -> ~A):

1. A <-> ~~B	Premise
2. (A -> ~~B) & (~~B -> A)	1 <->E
3. A -> ~~B	2 &E
4. ~B	* A
5. ~~~B	* 4 ~~I
6. ~A	* 3, 5 MT
7. ~B -> ~A	4-6 ->I

Sometimes we need to establish a conditional in order to establish the conclusion that we actually want. Here is an example of that situation wherein we are trying to show that (A -> B) -> C, R & B |- C v A:

1. (A -> B) -> C	Premise
2. R & B	Premise
3. B	2 &E
4. A	* A
5. B	* 3 Reit
6. A -> B	4-5 ->I
7. C	1, 6 ->E
8. C v A	7 vI

Let us now turn to ~I.